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Conjunction Probability Error

The conjunction rule applies to predictive judgment or forward conditional reasoning. It is a normative rule that states that the probability of any combination of events cannot exceed the probability of constituent events. For example, the probability of picking the queen of spades from a card deck cannot exceed the probability of picking a spade or a queen from the deck. Typically, people can successfully apply the conjunction rule to transparent problems such as the card selection problem. However, there is overwhelming evidence that when problems are less transparent, people often ignore the rule and judge the conjunction of events as more probable than a constituent event, thereby committing the conjunction probability error. Because of the pervasiveness the conjunction error and its clear violation of normative probability theory, it is important to understand conditions that tend to produce the error, procedures that may reduce its occurrence, and instances where it does not apply.

Conditions That Produce the Conjunction Error

The initial investigation of the conjunction error was conducted within the framework of under-standing how heuristic thought processes may produce systematic biases in judgment and choice. In their seminal investigation, Amos Tversky and Daniel Kahneman first explored the conjunction error as resulting from the use of the representativeness heuristic for judging probabilities. According to this heuristic, people judge probabilities for specific outcomes by making a similarity comparison with a model of the population from which the outcomes were sampled. For example, knowing that a person is a member of a particular group, one may use a stereotype of that group as a model to predict behaviors or attributes of the person.

The Linda Problem

An often used example that has been shown to produce robust conjunction errors is the *Linda problem*. As described by Tversky and Kahneman, Linda is 31 years old, single, outspoken, and intelligent. Participants are told that when she was a philosophy major at school, she was concerned with social justice and participated in protests and demonstrations. This background establishes a model of Linda as a sophisticated individual concerned with social issues. After reading the description, participants typically rank the relative likelihoods of predicted occupations and activities that apply to Linda. Three key statements that may be evaluated include the following:

(U) Linda is a bank teller.

(L) Linda is active in the feminist movement.

(U & L) Linda is a bank teller and is active in the feminist movement.

The first statement is unlikely (*U*) based on the model of Linda and is given a relatively low probability ranking. The second statement is likely (*L*) based on the model of Linda and is given a relatively high probability ranking. The third statement is the key statement as it conjoins the unlikely and likely events (*U & L*). As such, it represents a subset of both these events and cannot have a higher probability than either of these. Yet nearly all participants indicate that the conjunction is more probable than the unlikely event. These results are obtained with both statistically naive and statistically sophisticated participants and in situations in which participants are directly assessing the relative likelihoods of the events. Furthermore, a majority of participants still commit the error even when they are asked to bet on these outcomes, implying the effect does not disappear with monetary incentives for correct application of the conjunction rule.

The conjunction error in the Linda problem is constructed by pairing an unlikely outcome from the model with a likely outcome from the model. In the probability calculus, the probability of the combined events can be expressed as follows:

$$\Pr(U \,\&\, L) = \Pr(U)\,\Pr(L|U).$$

This formula makes it explicit that the probability of Linda being a bank teller and active in the feminist movement, Pr(*U & L*), must be less than or equal to the probability of her being a bank teller, Pr(*U*), as the probability of being active in the feminist movement given she is a bank teller, Pr(*L*|*U*), must be less than or equal to 1.0. But according to similarity-based heuristic thinking, combining an outcome that is dissimilar to the model with one that is similar to the model results in an evaluation of moderate similarity for the combined events. If probabilities are then based on such similarity evaluations, the mixed outcome case is judged as more probable than the unlikely constituent outcome, resulting in the conjunction error.

A Second Recipe for Conjunction Errors

The Linda problem used the recipe of combining an unlikely outcome with a likely outcome to produce the conjunction error. A second recipe for creating conjunction errors is to add an outcome that makes the other outcome more likely or plausible. Tversky and Kahneman illustrated this recipe in a health-related example in which participants were told that a health survey had been administered to a large sample of adult males of all ages and occupations. They were then asked to indicate which statement was more likely of a randomly selected person from the survey:

1. This person has had one or more heart attacks

2. This person has had one or more heart attacks and is over 55 years of age.

The majority of respondents chose the conjunction to be more probable in this instance. The specified age makes it easier for people to imagine this person having had one or more heart attacks. More generally, this type of conjunction error may be attributed to scenario thinking. In the first case, there is no reason to think that the selected individual might have had a heart attack. In the second case, the age-related information fills in some of the causal linkages that make the scenario more plausible and hence seem more probable. This type of scenario-based conjunction error can occur whenever a conjoined outcome provides a causal mechanism for the occurrence of the other outcome.

Application to Medical Decision Making

Tversky and Kahneman also demonstrated the applicability of the conjunction error directly to medical decision making. One of the problems they administered to two different groups of internists indicated that “A 55-year-old woman had pulmonary embolism documented angiographically 10 days after a cholecystectomy.” The doctors were asked to rank order the probability that the patient would be experiencing each of a set of conditions. These included “dyspnea and hemiparesis” and “hemiparesis.” Across the two samples, 91% indicated that the conjunction of conditions was more likely than the constituent condition. When physicians in an additional sample were confronted with their conjunction errors, they did not try to defend their decisions but simply indicated their surprise and dismay at having made such elementary errors. This last result suggests that the conjunction error is not simply due to misunderstanding how the alternatives are presented in the problem but instead represents a serious threat to risk assessment that can take place with experts within their own domain of expertise.

Procedures That May Reduce the Conjunction Error

Several criticisms of the work on the conjunction effect have been leveled over the 25 years since it was first reported. These criticisms focus on various features of how the problems are presented. One class of criticisms suggests that the problems may be ambiguously stated so that errors are due to participants misunderstanding what the experimenter is trying to communicate. For example, in several versions of the Linda problem, one simply chooses which is more probable, that “Linda is a bank teller” or that “Linda is a bank teller and is active in the feminist movement.” One might argue that the pragmatics of conversation norms lead individuals to interpret the first statement as meaning “Linda is a bank teller and is not active in the feminist movement.” Through the years, numerous ways of clarifying the options have been explored. The bottom line, however, is that although some versions may lead to fewer conjunction errors, they generally do not eliminate conjunction errors (i.e., the majority of participants still commit the error even with the reworded statements).

Another criticism has been directed against the normative force of the conjunction error. This argument is based on a strict frequentistic interpretation of probability, which states that it is reasonable to judge probabilities for samples from a population but it is not reasonable to judge probabilities for propensities of unique events. In the Linda problem, either she is or she is not a bank teller, and hence probability is not applicable. Rather than resolve this interpretation at the normative level, researchers have probed whether the conjunction error occurs when people are evaluating probabilities of samples from a population. For example, we can conceive of 100 women fitting Linda’s description and estimate the probability that a random sample from this population would have these characteristics. Although some studies have shown a marked reduction of conjunction errors in this case, most have demonstrated very strong conjunction errors still occur. The health survey example discussed above is one case in point.

Related to the issue of interpreting probabilities is the assertion that probabilities are not a natural way of processing frequency information and so people will make errors when forced to consider probabilities rather than frequencies. Several researchers have tested this idea by comparing performance on problems requiring probability assessments versus frequency assessments. Note that the frequency assessment requires that one talk about sampling from a population rather than talk about propensities of individuals. The health survey problem described above has been formulated in frequency terms by asking participants to estimate how many of a sample of 100 individuals from the survey would fit each description. In general, the response format of estimating frequencies sampled from a large population has led to a significant reduction of conjunction errors, with the majority of participants not committing the error. This method would then appear to be a good way to reduce reasoning errors and de-bias judges.

However, a closer look at the pattern of results across numerous studies indicates that it is not the frequency format itself that is strongly reducing conjunction errors; rather, it is the requirement of making estimates that is critical. Numerous studies have shown that choosing which alternative would result in the highest sampled frequency does little to reduce conjunction errors. It is only when estimates must be generated for each option that conjunction errors are dramatically reduced. This occurs even when the estimates are of probabilities rather than of frequencies. This result supports the idea that people have at least two distinct ways to process probability information. One may be more qualitative and heuristic-based and the other more numerical and algorithmic. When the response mode is qualitative in nature, as in ranking and choice, people tend to apply the qualitative heuristic mode of thought and commit conjunction errors. When the response mode requires numerical assessments, people are more inclined to apply the quantitative algorithmic approaches and hence reduce conjunction errors.

Applicability of the Conjunction Rule

It is important to note when the conjunction rule does and does not apply when considering the various tasks associated with assessing probabilities. The conjunction rule applies to predictive judgment or forward conditional reasoning. In this type of reasoning, events are conditioned on a premise represented as a hypothesized model or hypothesized sampling procedure. In the medical decision-making context, it applies to predicting symptoms given a disease or outcomes given a procedure. In these cases, one must be careful to consider whether probability assessments are being inappropriately increased by the consideration of a conjunct that makes a particular outcome easier to envision. It is important to avoid scenario thinking or similarity-based thinking in making these assessments.

The conjunction rule does not apply to diagnostic judgment or backward conditional reasoning. In this kind of reasoning, one is inferring the probability of a hypothesis based on an outcome or a conjunction of outcomes. In medical decision making, this is by far the more common type of assessment. Given a particular set of symptoms one must estimate the likelihood of a given disease as the cause. Here, Bayesian updating applies so that conjoining a diagnostic symptom with a nondiagnostic symptom should lead to an increase in the overall probability of the disease. One possibility is that people commit the conjunction error because they do not correctly differentiate between these two tasks and hence incorrectly apply diagnostic reasoning to a prediction task.

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See also Bayesian Evidence Synthesis; Biases in Human Prediction; Frequency Estimation; Heuristics; Probability Errors

Further Readings

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