Ratio Measurement and Theoretical Inference in Social Research*

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ABSTRACT

This paper attempts to rectify false suspicions that have developed among some regarding ratio correlations. If researchers are theoretically concerned with relationships between ratio variables that contain common components (e.g., population density and suicide rate) or between a ratio variable and one of its components (e.g., percent urban and population size of nations) correlation analysis will not normally yield spurious results. Spuriousness becomes a problem primarily when the researcher is interested in relationships between component variables which are standardized by a common term for reasons extraneous to the hypothesis. Even here, though, it is not the empirical correlation that is spurious, but possible inferences one might draw about the relationship between the component variables from the observed correlations between the ratio-standardized variables. Assessment of suggested alternatives to ratio correlation indicates that ratio measurement sometimes provides conceptual, interpretive, and statistical advantages over alternatives critics have proposed.

Because uncritical acceptance of faulty research poses a constant danger in the social sciences, we are routinely reminded to be diligent in evaluating possible methodological shortcomings in empirical studies. But an equally serious problem arises when criticisms of existing methodological procedures are automatically accepted without similar evaluation and reflection. Published methodological critiques not only often influence the type and direction of future methodologies used in studying a phenomenon, but they also can have a back-to-the-drawing board impact that leads many readers to discount the findings of previous studies and initiate new

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(and possibly superfluous) waves of research aimed at studying the same phenomenon (MacMillan).

This is the case with criticism of one of the most common forms of social science measurement, ratios and proportions. Building on the pioneering work of Karl Pearson, several writers have suggested that serious problems may arise when one correlates ratio variables that contain common terms in their numerators or denominators, or when a ratio variable is correlated with one of its component terms (Freeman and Kronenfeld; Fuguitt and Lieberson; Schuessler a, b). Common terms, it is argued, may artificially inflate or deflate measures of association between variables depending on the location of the common term in the ratios. The implication is that many statistically significant associations between ratio variables reported in the literature are merely artifacts of correlating variables with common terms and, hence, may be more spurious than real.

Some critics have been careful to point out that correlations between ratio variables containing common terms are valid if a theoretical interest lies in the ratio constructs (Schuessler, a, b). But many readers have overlooked or ignored such qualifications. Indeed, there is mounting evidence that those articles criticizing ratio correlation may be creating more of a problem for social research than their authors anticipated. Numerous studies that appropriately used ratio correlations have been drawn into question (Kimberly); journal referees are rejecting or calling for major revisions in research submitted for publication primarily because the statistical analysis contains correlations between ratio variables with common terms; and many theoretically interesting relationships tapped by ratio measurement are not being investigated by researchers for fear of generating empirical tautologies.

What makes the situation all the more disturbing is that as knowledge of possible problems of ratio correlation spreads through the professional audience, the use of ratios and proportions is increasingly criticized by people who are aware of the literature dealing with the issue, but who are unfamiliar with the details of the arguments and the important qualifications that have been made. As a result, ratio and other compound measures of social patterns and system properties are being rejected out of hand, only to be replaced with other measures that are sometimes conceptually and statistically inferior.

Our purpose is to reexamine the ratio correlation issue and, in so doing, explicate its theoretical utility and empirical validity as a form of analysis for many substantive problems. We will also attempt to rectify false suspicions that have developed among some regarding ratio and proportional measurement and alert researchers to potential problems of employing alternative measures and statistical procedures that have been suggested to replace correlations between ratio and other compound variables containing common terms.

Conceptual and Measurement Consistency

Stripped to its basics, the issue of the appropriateness of correlating ratio variables is theoretical rather than empirical. There is nothing inherently spurious or biased in the empirical correlation between ratios containing common terms (Kuh and Meyer; Schuessler, a). Even the classic ecological fallacy, which typically involves correlations between aggregated common-term ratio variables, is not an empirical fallacy, but one of incorrect theoretical specification and fallacious inference (Hanushek et al.). To use Robinson's well-known example, the negative correlation between nativity (foreign birth) and literacy at the individual level and the positive correlation between *percent* foreign born and *percent* literate at the territorially aggregated level are both statistically valid correlations representing social phenomena at their respective levels of analysis. If one's hypothesis were that foreign born predominate in territorial units where literacy rates are high, then the correlation between percent of the population foreign born in the territorial units and percent literate would be an appropriate statistical test of that hypothesis.

Regrettably, the notion that common terms, by definition, lead to spuriously positive or negative statistical associations has found its way into and established itself in the literature. Nowhere is this better illustrated than in criticisms of studies that have examined the effect of organization size on administration intensity and internal span of control in bureaucratic institutions (Akers and Campbell; Freeman and Kronenfeld; Kimberly). A mathematically built-in negative bias is said to exist when one correlates organization size (measured by total number of personnel) with administrative intensity measured by the ratio of administrative personnel to total personnel or to production personnel). Akers and Campbell describe the problem as follows:

> It is somewhat tautological to relate changes in the relative size of the staff component to total organizational size when the administrative component is calculated as the ratio of staff to total membership. One is attempting to measure the association between X and Y, when the measurement of Y is a rate or index which includes X as the denominator. *This could lead to a spurious negative relationship since as X increases it provides an ever larger denominator which would tend automatically to decrease the value of Y.* (245; italics added)

Why the italicized portion of Akers' and Campbell's statement would hold escapes us, since the percentage or ratio of an organization's personnel resources allocated to administrative functions may increase, decrease, or remain the same as the organization expands in size predicated, ceteris paribus, on organizational imperatives and the organization's corresponding requisites for supervision, integration, and communication. These requisites establish the causal conditions linking changes in the number of administrators relative to changes in the number of production personnel as the organization expands in size. Without knowledge of those conditions, nothing can be stated a priori about the direction of the relationship between organization size and administrative intensity (MacMillan).

Nevertheless, Freeman and Kronenfeld through the use of calculus and computer simulation claim to demonstrate that the common term has built mathematical dependencies into the size-administrative intensity relationship. MacMillan, and MacMillan and Daft show that Freeman and Kronenfeld's mathematical derivations are incorrect and provide the proper calculus. Moreover, if one carefully examines the conditions which Freeman and Kronenfeld establish to simulate their original values of A (number of administrators) and P (number of production personnel), one can see that the conditions rather than reformulation of the original values as ratios produce their reported negative relationship between administrative intensity and size.[1]

What Freeman and Kronenfeld *do* demonstrate is that there may be no correspondence between the correlation of number of production personnel with *number* of administrators and the correlation of number of production personnel with the *ratio* of administrative to production personnel. Thus, they observe that it is impossible to infer anything about the former relationship from empirical knowledge of the latter. This is certainly true as has been documented repeatedly since Pearson's article in which he labeled the disparity between correlations of basic component variables and correlation of their ratio transformations "spurious correlations."

It is important to emphasize, however, that in studies of the effects of organization size on administrative intensity or span of control, the problem of drawing inferences about the magnitude and direction of the relationship between organization size and *number* of administrators or *number* of production personnel is not at issue. Administrative intensity (measured by the percentage of an organization's personnel devoted to administrative activities) and span-of-control (measured by the ratio of administrators to production personnel) are structural properties of organizations that are conceptually distinct from either the absolute number of administrators or the absolute number of production personnel.[2] Indeed, if organizational structure expanded with constant linear scale, the expected correlation between size and either the absolute number of administrative personnel or the absolute number of production personnel would be +1.0, while those between size and ratio-defined administrative intensity or span-of-control would be 0.0. There is nothing spurious about either set of correlations, although any inferences drawn about relationships among an organization's raw component variables made from observed correlations among its structural variables might well be spurious (Fuguitt and Lieberson; Kuh and Meyer).

An important conceptual distinction between types of system change is pertinent here; when recognized, it clearly indicates an important and proper use of ratio measurement. This is the conceptual distinction Spencer made between growth and development, or Boulding's analogous conceptual distinction between *simple* growth and *structural* growth. Simple growth refers to the accretion or depletion in raw component variables (e.g., the number of administrators or the number of production personnel), whereas structural growth (which may also be negative) refers to alteration of the component variables relative to one another. Thus, organizations may grow (in the manner of Boulding's simple growth) without changes in their internal structure (scale) as long as the ratios or proportions of their component variables remain constant.

It is the hypothesis relating size to nonproportionality in structure of social systems that some researchers have attempted to test through the use of ratio variables (e.g., Haire; Kasarda; Nolan, b). This hypothesis as Boulding articulates it states:

> As any structure grows, the proportions of its parts and significant variables *cannot* remain constant. It is impossible, that is to say, to reproduce *all* the characteristics of a structure in a scale model of different size. This is because a uniform increase in the linear dimensions of a structure will increase all its areas as the square, and its volumes as the cube, of the increase in the linear dimension. Thus a twofold increase in all lengths of a structure increases its areas by four times and its volume by eight times. As some essential functions and variables of structure depend on its linear dimensions, it is impossible to keep the same proportions between all significant variables and functions as the structure grows—This is the basic principle which underlies the "law of eventually diminishing returns to scale" familiar to economists (374).

Most would agree that ratio variables tap proportionality in organizational structure extremely well. In fact, it might be argued that the epistemic correlation between the concept of proportionality and its ratio measure is near 1.0, since the concept is typically defined in terms of the measure. This may seem an obvious statement, but sight of the obvious is frequently lost in the maze of mathematical symbols and assumptions contained in critics' arguments.

A related rationale for using ratio variables to tap basic structural dimensions of macrosocial units is to adjust for the simple aggregative effects of unit size (or population at risk) for the phenomena under study. Without adjusting for these effects, much comparative analysis of social phenomena aggregated by units of different size would be trivial or uninteresting. For example, some argue that city size has important consequences for a broad range of social patterns ranging from crime to economies of scale in the provision of public services (Alonso; Ogburn and Duncan). If one were concerned with the consequences of city size for crime, little beyond the obvious would be confirmed by demonstrating a significant positive correlation between city size and absolute number of crimes. The theoretically more interesting question is whether the relative propensity for crimes or, correspondingly, the probability that a resident will be the victim of a crime, varies with city size. The measurement that taps this propensity or probability dimension is the crime rate. Crime rate is a structural property of the collective that defines the relation between two of its component elements (i.e., crimes per member). It is conceptually distinct from either component and its correlation with city size is both theoretically informative and empirically valid.

On the other hand, the correlation between city size and absolute number of crimes, although empirically valid, is not particularly interesting. Larger places tend to have larger absolute amounts of most phenomena—more rich, more poor, more employed, more unemployed, more doctors, more deaths, more cops, more crimes. This is because the aggregated absolute amounts reflect not only the underlying structural relation between elements of the collective but also the population at risk. In fact, continuing with our crime example, the absolute number of crimes that occur in a city may be treated as the algebraic product of its theoretically more interesting structural property (the probability that a resident will be a victim of a crime) and population at risk (number of residents). Similar formulations hold for other aggregated social phenomena where absolute number of events occurring (E) can be treated as an algebraic product of its probability of occurrence (E/P) and population at risk (P). Dividing absolute number of events ($E/P \times P$) by population at risk (P) may be viewed as an algebraic adjustment for population at risk that enables one to tap directly the underlying structural property (Lyons). Indeed, because population at risk is such an important analytical component in the absolute number of occurrences of so many aggregated phenomena, one could turn critics' arguments completely around and claim that without standardizing these aggregated phenomena for population at risk, one may be tautologically correlating variables that simply reflect this common analytical component.

For similar reasons, in assessing economies of scale in the provision of municipal services, the theoretical question is not whether total operating service costs increase with city size. Rather the question is whether operating costs increase at a faster rate than population. Correlating city size with per capita costs of public services, ceteris paribus, is an appropriate statistical assessment of this question. The procedure does it quite simply and effectively. Constant per capita costs across cities of different size indicate neither economies nor diseconomies of scale. If per capita costs increase, ceteris paribus, there are diseconomies of scale; if the per capita costs decrease, there are economies of scale. Moreover, the metric slope and correlation coefficient between city size and per capita service costs provide useful indicators of the magnitude of the scale effects and amount of variance in economies or diseconomies accounted for by size differences.

Thus far we have chiefly treated the correlation of a ratio variable with one of its components. There are, of course, many research questions where the theoretical issue directs one to correlate two or more ratio variables that have common terms. For example, one may wish to examine the effects of rates of community growth on rates of suicide, where the denominator of both structural variables is the community's mid-term population size. Similarly, one's hypothesis may be that net migration rates for territorial units are related to their unemployment rates, industrial structure (measured by percent employed in selected industries), and population composition (percent of resident population in various income or racial categories). Or, organizational theorists may wish to examine the relationship between internal spans of control (as defined previously) and the mean skill level of production workers, and then relate both of these variables to output (or value-added) per production worker. In all such instances, the issue of spuriousness does not arise, as long as the hypothesis under study and inferences drawn relate to the ratio constructs.

If correlations between compound variables containing common terms are not empirically biased, what is the reason for continued skepticism of their use in sociology? We submit that the problem is, again, conceptual. Rates, ratios, proportions, percentages, and measures of the composition and distribution of various phenomena (any combination of which may contain common terms) typically refer to structural properties of groups or collectives. Curiously, we find that most sociologists do not employ collective or group properties in building their theories.[3] There remains a powerful tendency in the profession to reduce the study of social phenomena to the individual or smallest measurable component. This orientation fosters the view that compound structural variables have no intrinsic theoretical meaning and that the real answers or true relationships are to be found only at the individual or disaggregated level of analysis.

Thus, for example, we find Fuguitt and Lieberson amending their qualification that ratio correlations are nonproblematic when theoretically justified, with an epistemological judgment:

> First an argument can be made that spurious correlation is not an issue in correlating ratio or difference terms, provided that one's interest is exclusively in the composite variables rather than in the components. We believe, however, that it is usually difficult to maintain that position; problems can be reformulated in terms of component variables, or in any event the relation between components and composite variables may be profitably explored (141).

Fuguitt and Lieberson's belief corresponds to the widely held notion alluded to above that one cannot be legitimately interested in relationships between compound structural variables, sui generis. According to this misapprehension, if the compound variables can be empirically decomposed into their components, the problem can and should be conceptually and statistically reformulated in terms of the components, otherwise the study will likely yield spurious results. (Bollen and Ward).

Much of the false suspicion of spuriousness when correlating aggregate compound variables can, of course, be traced to Robinson, who claimed that correlations between such variables are usually, if not always, computed as inferior substitutes for theoretically preferable individual level measures. As respondents to Robinson have long pointed out, however, correlations between compound variables that measure properties of collectives or areal units are frequently not computed merely because individual level measures are unavailable (Duncan et al.; Lazersfeld and Menzel; Menzel). Theoretically meaningful questions may be posed at the level of the aggregate (many addressing relationships between ratio-defined properties) and theoretical specifications that hold at one level may not hold or even be pertinent at another level (Hannan). Moreover, there are many kinds of compound variables that measure properties of collectives or areal units that do not have individual level counterparts, or whose conceptual meaning is substantially altered when decomposed and analyzed as separate components. Examples include population and network density, occupational or industrial diversity, urban primacy, net migration, most system rates (e.g., death rate), and various measures of the spatial distribution and demographic composition of system subunits. For some research problems, it is the common term in the composite measures that makes it possible to test the theoretical specification. Thus, land economists have effectively correlated composite indices with common term components to answer such questions as: Do locational advantages with respect to commodity A or activity C lead to locational advantages with respect to commodity B or activity D? Likewise, building on the classic propositions of Durkheim, Simmel, and Wirth, sociologists may wish to assess the empirical relationship between population density and density of social interactions, or between population density and network density. For such problems, correlations between compound variables with common terms are not only theoretically justified, but are essential to uncovering certain empirical relationships between the defined structural properties.

How Satisfactory Are Alternative Methods?

Even assuming that one could decompose compound structural variables and work solely in terms of their components without loss of conceptual meaning (an assumption which we obviously question), statistical problems are not solved. On the contrary, we find indications that alternative methods have statistical shortcomings which make them even less effective than the ratio methods they would replace.

For example, in the literature on organizations, two general methods have been proposed as alternatives to ratio measurement in investigations of the effect of organization size on such concepts as administrative intensity, administrative overhead, economies of scale, and nonproportionality. The first is to regress the number of administrators on the total number of employees (or number of production personnel) and examine the Y-intercept. Freeman and Kronenfeld endorse this method:

> If A is regressed on P and the A-intercept is other than zero, a disproportionate relationship exists between A and P. When it is less than zero, A increases disproportionately *rapidly* relative to P. If, on the other hand, the A-intercept is greater than zero, A increases disproportionately *slowly*. In the latter case economies of scale may be said to exist (118).

The second method, originally proposed by Akers and Campbell (and also discussed by Freeman and Kronenfeld), is to convert number of administrators and total number of personnel to logarithms and regress the logarithm of A on the logarithm of P. If the unstandardized slope is other than 1.0, a nonproportional relationship is said to exist. A coefficient larger than 1.0 indicates that administrators increase at a faster rate than organization size, whereas a coefficient under 1.0 indicates that number of administrators grows more slowly than size.[4]

Thus we are offered two other procedures for detecting nonproportionality and administrative economies of scale which are not plagued by the so-called problem of definitional dependency. MacMillan, and MacMillan and Daft demonstrate that these procedures are analytically equivalent to the use of the administrative ratio, and, in general, we agree. However, when one is confronted with actual data, these alternatives have serious limitations of their own.

The chief drawback of the first method is its pronounced sensitivity to extreme values. One or two extreme values can quite easily draw the intercept of the regression equation above or push it below the origin. Moreover, as even Freeman and Kronenfeld warn, the amount of departure from the origin is not a valid indicator of the *extent* of nonproportionality in the relationship between A and P.

The second method also has shortcomings in certain research contexts. When one converts A and P to logarithms, theoretical statements and interpretations of observed relationships become more complex, particularly if one's model is multivariate. Thus, as Freeman and Kronenfeld (119) observe, if other variables in the model (such as production technology) have additive effects on administration, the log transformations of A and P imply an inappropriate model, whereas conversion to a full multiplicative model may result in specification error or false inferences.

Since the Y-intercept from raw variable correlation may be dominated by a few extreme values whereas logarithmic transformation of the raw variables tempers the effects of extreme values (i.e., outliers are drawn more toward the origin), there is a possibility that the alternative methods may give contradictory results. This, indeed, has been found to occur in one of our own studies (Nolan, a) as is illustrated in the top panel of Table 1. Examining the effect of population size on the administrative component of large nations, we can observe that the positive raw Y-intercept would indicate that the relative scale of government employment is a *decreasing* function of population size, whereas the log slope of 1.292 would indicate the opposite—that it is an *increasing* function of population size. Testimony from the two measures is reconciled only when two outliers (India and the United States) are removed. With the outliers removed, both the log slope and the raw intercept indicate that the scale of government employment is an increasing function of population size. In support of the ratio construct, the second panel of Table 1 shows that, regardless of whether the outliers are included or removed, the correlations between population size and percent employed in government (and different combinations of log transformations of these measures) are all positive.[5]

Another limitation shared by both alternatives to ratio measurement of administration intensity is that correlation coefficients generated by regressing the number of administrators on the total number of personnel (or their logarithmic transformations) suffer from the part-whole correlation problem (Mueller et al., 272). The correlation coefficients will tend to be

Table 1. COMPARISON OF CORRELATION AND REGRESSION METHODS FOR ANALYZING THE RELATIVE SIZE OF GOVERNMENT ADMINISTRATION IN NATIONS WITH POPULATIONS GREATER THAN 5,750,000

	All Nations N = 35	Ourliers Removed N = 33
Logarithmic Slope	1.292	1.352
Raw Y-Intercept	149702.609	-7520.900
	Correlations of Ratio Variables	
Population with percent in government	.11	.14
Log of population with percent in government	.16	.22
Population with log of percent in government	.26	.21
Log of population with log of percent in government	.32	.29

artificially high and not provide a suitable indication of the amount of variance in administrative intensity or spans-of-control explained by organization size.[6] On the other hand, regressing administrative ratios or percentages on organization size provides easily interpretable metric slopes of the relationship between organization size and administrative intensity or span-of-control as well as an excellent indication of the amount of variance in this structural property accounted for by organization size.[7]

As a final caveat about alternative procedures, let us briefly address the issue that correlations between macro-system properties such as percent urban and per capita energy consumption are spuriously inflated because they have a common denominator. This problem, which all critics note, would seem to have ominous implications for the bulk of cross-national studies that have used standardized measures as well as for any future comparative analysis of properties of social systems that are of different population size or land area.

As we have repeatedly stressed, if one's hypothesis is formulated conceptually in terms of ratio (e.g., per capita) measures, spuriousness is not an issue. However, if theoretical interest is primarily in the relationships between the numerator series which has been standardized through division by terms extraneous to the hypothesis, spurious inferences could be drawn. To avoid standardization bias in analyses of a theoretically specified numerator series, some researchers suggest that instead of dividing the variables of interest by population size or land area, it might be better to regress each of the original variables on population size or land area and work with the residuals in the correlation or multiple regression analysis.

We assessed this suggested alternative with a comparative data set containing indicators of contact technology and general economic development for nation states, along with measures of their population size and land area for which we wish to standardize. We found, first of all, that the residualization procedure led to greater complexities in interpreting correlations and metric slopes between residualized terms as compared to the relatively straightforward interpretations of ratio correlations and slopes between terms standardized on a per capita or per unit land area basis. Moreover, as the correlations presented in Table 2 show, the residualization procedure actually results in higher correlations between most indicators than does the traditional ratio standardization procedure. This finding is not surprising when we realize that the high correlation between the indicators in Table 2 is likely due to their reflection of a common macro dimension (e.g., societal scale, technological development, national wealth), rather than a statistical artifact of division by a common term.[8]

Closely akin to correlating residuals is the suggestion that if theoretical interest lies in the relationship among absolute values of aggregated phenomena, but that the aggregated phenomena themselves are functions of population size or land area, one should employ a multiple regression analysis using the absolute values, with population size or land area in-

Table 2. INTERCORRELATIONS OF MEASURES OF CONTACT TECHNOLOGY WHEN STANDARDIZED BY TRADITIONAL RATIO AND RESIDUALIZED REGRESSION METHODS

	Traditional Ratio Method			Residualized Regression Method	
	RAILAR			RAILRES	
ROADAR	.17 N = 69		ROADRES	.95 N = 69	
	TELEPOP			TELERES	
MOPOP	.93 N = 69		MOTORES	.99 N = 69	
	MOPOP	TELEPOP		MOTORES	TELERES
GNP PER CAPITA	.84 N = 69	.89 N = 70	GNPRES	.97 N = 69	.89 N = 70

ROADAR = road miles per area
RAILAR = rail miles per area
ROADRES = road miles regressed on area
RAILRES = rail miles regressed on area
MOPOP = motor vehicles per capita
TELEPOP = telephones per capita
MOTORES = motor vehicles regressed on population
TELERES = telephones regressed on population
GNP PER CAPITA = gross national product per capita
GNPRES = gross national product regressed on population

cluded as an additional independent variable. Results of this procedure, however, can be seriously affected by a few extreme values and will often suffer from the damaging influence of heteroskedasticity (Kuh and Meyer; Theil). Perhaps worse, multicollinearity will often be so severe that it will frequently be impossible to obtain meaningful regression solutions.[9]

Overall, we have come to this position on alternative procedures of standardizing variables for system size in comparative analysis. If one's theoretical interest lies solely in the numerator series or absolute values of aggregate system phenomena, per capita and similar ratio standardization procedures may lead one to draw spurious inferences from observed correlations. However, since it is usually more complex to explain relationships between residualized terms than between per capita terms and because alternative procedures may exacerbate rather than reduce problems of extreme values, heteroskedasticity and multicollinearity, we recommend that researchers carefully consider possible limitations of suggested alternatives to ratio measurement before they abandon this traditional method of standardizing aggregated phenomena for system size.

Summary and Conclusions

Our aim has been to explicate important qualifications to criticisms of the use of ratio variables in social research. We have argued that the appropriateness of correlating ratio variables containing common components is a theoretical and not an empirical question. If researchers are theoretically concerned with relationships between ratio variables that contain common components (e.g., population density and suicide rate) or between a ratio variable and one of its components (e.g., percent urban and population size of nations), correlation analysis will not yield spurious results. Spuriousness becomes a problem primarily when the researcher is interested in relationships between component variables which are standardized by a common term for reasons extraneous to the hypothesis. Even here, though, it is not the empirical correlation that is spurious, but inferences that one is likely to draw about the relationship between the component variables from the observed correlation between the ratio-standardized variables. For hypotheses formulated theoretically in terms of rates, proportions, percentages, or amounts per capita, correlations between and among these ratio variables provide unbiased estimates of the structural relationships.[10] It should be stressed, however, that, to avoid possible pitfalls that critics have pointed out, one must insure that hypothesis conceptualization, variable measurement, and inferences drawn from observed statistical relationships are consistent.

We have also examined the argument that, by definition, there is a built-in negative correlation between such variables as organization size and the proportion of the organization's membership performing administrative functions. Close scrutiny of the critics' analysis supports the contention that the negative correlation is built into the relationship by *assumptions of the critics*, rather than from any inherent tautological property of the measures.[11]

Finally, we empirically assessed suggested alternatives to ratio measurement of structural properties of organizations and other social systems. Our comparative analysis indicates that in some research contexts traditional ratio variables exhibit conceptual and interpretive, as well as specific statistical advantages over suggested alternative measures and procedures.

Notes

1. In a revealing note, Freeman and Kronenfeld point out that by simply manipulating these conditions they were able to generate correlations varying from −.999 to +1.00 between size and administrative intensity. This would seem to provide further confirmation of MacMillan and Daft's conclusion that it was the assumptions under which values of A and P were generated by Freeman and Kronenfeld instead of mathematical dependencies that led to the observed results.
2. We follow the standard definition of a structural property as that which refers to the relation between differentiated roles, elements, or subunits in a system (Riley). Structural properties have also been defined as "properties of collectives which are obtained by performing some operation on the data about the relations of each member to some or all of the members" (Lazarsfeld and Menzel).
3. In a survey of major journals of sociology, Brown and Gilmartin found a surprising concern

with the state of individuals. In fact, they found that the individual was the unit of analysis in the majority of studies.
4. This double-logarithmic slope has been used to measure system allometry (relative growth of parts as compared to the whole) in a variety of biological and social systems (Svalastoga) and is equivalent to the econometricians elasticity coefficients (Johnston).
5. These positive correlations are consistent with the findings of previously published cross-national studies that examined the relationship between size and the administrative component of nation-states using different procedures and administrative component measures (Kasarda; Hofstatter cited in Svalastoga).
6. Whereas regressing number of administrators on number of production personnel will not be a part-whole correlation, their bivariate correlation will usually be very high since they both reflect organization size. Moreover, the correlation will not be a suitable indicator of variance explained in administrative intensity or span-of-control by organization size.
7. Ordinary least squares estimation procedures can also be used to examine and provide substantive interpretations of size-administrative ratio relationships under a variety of non-linear associations between such terms as those represented by polynomial or second degree quadratic equations (Stimson et al.).
8. It is also important to point out that the above form of residualization will not normally yield a true least squares regression solution. Draper and Smith, who discuss the procedure under the rubric of stage-wise regression, show that the regression estimates (b values) will be biased to the degree that the residualizing (standardizing) terms are correlated with the numerator series.
9. Procedures such as ridge regression have been developed to deal with problems of multi-collinearity (highly correlated independent variables), but it should be noted that ridge regression procedures also produce biased estimates (Hoerl and Kennard, a, b; Kasarda and Shih).
10. Correlation of ratio variables is, of course, subject to the same strictures about measurement error and specification error (i.e., omitted explanatory variables) as is correlation of nonratio variables. Measurement error in component variables and omission of pertinent explanatory variables can bias the results of ratio correlations just as they can bias results from other types of correlated variables.
11. Space constraints precluded our discussion of similar attempts in the literature to demonstrate dependency bias in correlations between ratio variables with common terms. Suffice it to say here that critics assumed that theoretical interest actually rests in the components rather than the ratio variables per se, and, having assumed so, they proceed to demonstrate the severity of bias through ratio standardization procedures so unrealistic or extreme that we can imagine only the most naive researcher employing them. Such procedures include standardizing (by dividing) two completely uncorrelated variables by a third variable that is also uncorrelated with the other two, or weighting two per capita measures by population size and correlating the weighted results. In either case, critics are making statements about their assumptions and unrealistic procedures rather than about the seriousness of problems engendered by the informed use of ratio variables.

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