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How element visibility affects visual enumeration

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The task of detection requires that at least one target component (i.e. "feature") be detected, while the task of identification requires the detection and integration of multiple features into a recognizable object. Enumeration seems to involve aspects of both feature detection and object identification. As in identification, it requires the detection of multiple features, but as in detection, it does not require the explicit encoding of a global form. Enumeration of briefly presented objects is accurate up to the "subitizing capacity" of 3–5 items. We discuss the relation of enumeration to visual detection and identification by considering the effect of target visibility on subitizing capacity. We found that while the distribution of enumeration responses changes with contrast, subitizing capacity is generally invariant with contrast until it nears detection threshold. These results suggest that component detection (associated with number estimation) and component integration (associated with subitizing) behaved differentially as contrast was manipulated. We speculate that subitizing capacity is linked to the approximate number of detected features adequate for recognizing shapes.

1. Introduction

Historically, vision science has been concerned with the limits of vision, such as the faintest light that can be detected or the smallest letter that can be identified. Here, we are interested in the limits of visual enumeration and, more specifically, in the highest number of elements that can be precisely apprehended at a glance. Exploring the processes underlying visual enumeration may uncover mechanisms that intersect number representation, perceptual organization, attentional selectivity and working memory.

Most enumeration studies have found a discontinuity between enumerating small and large numbers, which has been taken to reflect two separate cognitive mechanisms. Enumeration of about four or fewer elements is fast and precise, and has been referred to as "subitization" (Kaufman, Lord, Reese, & Volkmann, 1949). Enumeration of about five or more elements has been labeled as counting or estimating. If there is sufficient time, observers may *count* each item slowly and serially; if not, they may quickly and imprecisely *estimate* the number of items in parallel (Dehaene, 1992).

One prominent theory of subitization is the FINST (fingers of instantiation) model, introduced by Pylyshyn and colleagues (e.g., Pylyshyn, 1994; Trick & Pylyshyn, 1993, 1994). It proposes that enumeration within the subitizing range is mediated by a limited-capacity index that tags individual items without the need for focused attention. When the number of items to be enumerated exceeds the FINST capacity, then attention must be deployed serially to multiple spatial locations. Others have proposed a similar limited mechanism within short-term working memory capacity (Cowan, 2001), spatial frequency filters (Atkinson, Campbell, & Francis, 1976) or polygon formation (Mandler & Shebo, 1982).

Despite the accumulated evidence in favor of the idea that enumeration is handled differently within the subitizing range (i.e., a discontinuous enumeration function), the issue is not entirely settled. For example, one of the proposed hallmark of subitization is its "preattentiveness" (Trick & Pylyshyn, 1993, 1994). Yet more recent evidence suggest that enumeration both within and beyond the subitizing capacity is modulated by attention (Burr, Turi, & Anobile, 2010; Egeth, Leonard, & Palomares, 2008; Olivers & Watson, 2008; Railo, Koivisto, Revonsuo, & Hannula, 2008).

Thus it is possible the enumeration function might be a continuous nonlinear function that only involves a single cognitive process. Van Oeffelen and Vos (1982) observed that although the differences between successive numbers are constant (e.g. 2–1; 6–5), the ratios (e.g. 1/2; 5/6) between successive numbers decrease as the number of objects increases following a power function with an exponent of -1 (see also Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). This observation underscores the possibility that enumeration performance may decrease in a nonlinear way. Furthermore, thorough analysis of reaction time distributions for enumeration resulted in no significant discontinuity for enumerating small and large numbers (Balakrishnan & Ashby, 1991, 1992). Aside from behavioral work, brain-imaging studies have also been

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inconclusive. Using PET, for example, Sathian et al. (1999) found that subitizing and counting have different cortical networks while Piazza, Mechelli, Butterworth, and Price (2002) found otherwise.

Previous enumeration studies have mostly used highly visible elements, making them susceptible to a ceiling effect that could result in flat accuracies in the subitizing range. A classic study on enumeration accuracy as a function of presentation time and luminance resulted in a discontinuity in the enumeration function at eight elements (Hunter & Sigler, 1940). We extend this study by further controlling for element visibility. First, we display our elements along a single eccentricity to maintain more uniform visibility, as elements that fall closer to the fovea might be better enumerated. Second, we control for changes in brightness in our display because we manipulate contrast (Michelson definition) rather than luminance. Luminance presents a confounding cue, since observers might potentially use overall display brightness as a substitute for number. Third, we account for visual crowding (Pelli, Palomares, & Majaj, 2004) by ensuring that elements were displayed beyond the critical spacing of crowding of about one-half of viewing eccentricity (Bouma, 1970).

The purpose of this paper is to evaluate how stimulus visibility affects enumeration accuracy and variability, and how independent detectability of individual elements might explain the data. Here, we used the rules of probability summation to characterize feature detection, against which enumeration functions can be compared (Schlosberg, 1948). In particular, we assessed how luminance contrast affected subitizing capacity, an empirical discontinuity between small and large numbers in enumeration functions.

1.1. Accuracy of detection probability

The logic behind the probability model in accuracy data is as follows: enumeration can be described as the simultaneous detection of multiple items because the observer is required to precisely detect how many elements are present. This model simply asks, "What is the probability of detecting n elements given that n elements were presented", *p (detect n elements|n elements presented)?* If probability governs enumeration, then the probability of detecting the exact number of gratings decreases exponentially as the number of items is increased, $p = p_1^n$ (Fig. 2b), where p_1 is the probability of detecting a single grating and n is the number of presented gratings. According to this model, the probability of precisely detecting multiple elements can be predicted from the probability of detecting a single element.

If enumeration adheres to this probability model, it would undermine the notion that subitizing and counting are two separate processes, and would instead reflect two distinct levels within a continuum of difficulty (Balakrishnan & Ashby, 1991, 1992). It would further suggest that the computation underlying enumeration is a single mechanism based on the probability of detecting a single element. Fig. 2b shows that the functions predicted by probability change shape dramatically. For low contrast gratings, the enumeration function is concave, and accuracy drops almost linearly for small numbers then plateaus for large numbers. For high contrast gratings, the enumeration function is convex, and accuracy remains high for small numbers then drops quickly for large numbers. In this case, data generated by an underlying exponential process where p_1 is high might look suspiciously like subitizing, as the function for low numerosities would be near ceiling.

1.2. Response variance of detection probability

Variability is another feature of our data that can diagnose the role of probability summation in visual enumeration. If enumeration is governed by independent detection of individual elements then the distribution of responses for a given number should follow the rules of probability. That is, as more elements are presented, response variability decreases as a function of the square root of the number of presented elements (Cordes, Gelman, Gallistel, & Whalen, 2001; Gallistel & Gelman, 2000). If this were the case, then the coefficient of variation (standard deviation/mean response) plotted as a function of mean response would have a log–log slope of −0.5 following the properties of a binomial distribution (see Appendix B). Alternatively enumeration may follow a scalar pattern – standard deviation would be proportional to element number such that the coefficient of variation as a function of mean response would be flat, a slope of 0. Many studies have found that coefficient of variation is constant as a function of number for numerosities greater than the subitizing capacity (e.g., Nieder & Merten, 2007). The coefficient of variation is a measure identical to a Weber fraction, in which precision for performing a task is normalized relative to the magnitude of a stimulus parameter – number of elements in this case (see Burr et al., 2010; Feigenson, Dehaene, & Spelke, 2004; Nieder & Merten, 2007).

2. Methods

2.1. Participants

Eighteen observers participated in this experiment (18–27 years old). All had normal or corrected to normal vision. Observers were given course credit or monetary compensation for participation.

2.2. Stimuli, apparatus and procedure

This experiment was executed on an Apple iMac G3 computer attached to a 19″ NEC monitor using MATLAB software with the Psychophysics Toolbox extensions (Brainard, 1997; Pelli, 1997). The gray background luminance was ~18 cd/m^2, the middle of the monitor range. To ensure equal visibility across targets, we controlled viewing eccentricity and inter-target separation. Targets were presented at 12 possible evenly spaced locations each 5° from fixation. The minimum distance between targets was 2.6°, target-center to target-center.

The targets were Gabor patches (i.e. black and white patterns with a sinusoidal luminance profile; Fig. 1) that were randomly oriented 0°, 45°, 90° or 135°. The Gabor patches had a spatial frequency of 1 c/deg. and had a width of 0.52°, which is the 1/e radius of the circularly symmetric Gaussian envelope of the grating. There were five experimental blocks corresponding to five Michelson contrasts $[(L_{\max} - L_{\min})/(L_{\max} + L_{\min})]$: 0.12, 0.18, 0.25, 0.50 and 0.96, where L = Luminance values. Each experimental block had 120 trials. Before the experimental blocks, observers performed 10 practice trials.

Observers fixated on a 0.15° black square at the center of the screen, which was displayed for the entire trial. The stimuli were presented for 50 ms and viewed binocularly. Observers used a chin rest and sat 60 cm away from the screen. Observers enumerated how many gratings were presented on the screen. For each trial, the number of elements was randomly chosen. There were 10 possible answers: 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9 gratings, which were typed on the keyboard. Correct answers were rewarded with a short beep. Because of a response bias in which observers disproportionately choose ordinal extremes, only data for 1–8 gratings were analyzed.

3. Results and discussion

In this study, accuracy and variance of enumeration were evaluated at different contrasts to assess the role of element visibility in enumeration. It specifically tested the possibility that enumeration performance can be predicted from probability summation of detecting independent elements (see Schlosberg, 1948). In the fol-

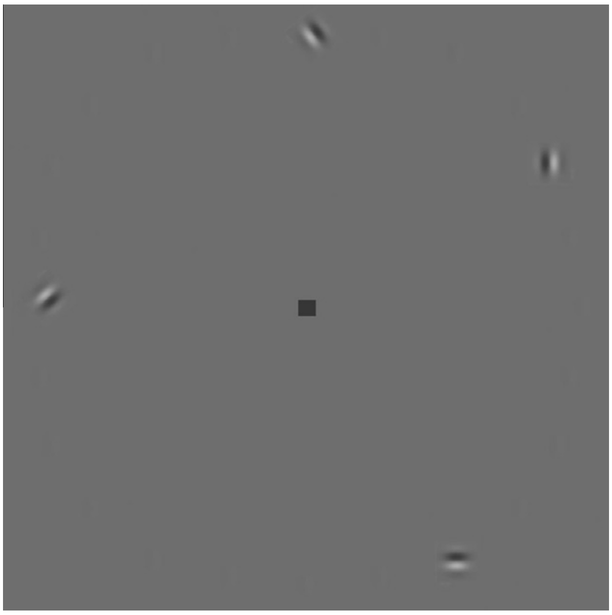

Fig. 1. Example of stimuli. Targets were sinusoidal Gabor patches that were independently oriented 0, 45, 90 or 135 deg. They were presented at 5 deg viewing eccentricity along an imaginary circle for 50 ms. Observers enumerated how many gratings were presented from 0 to 9.

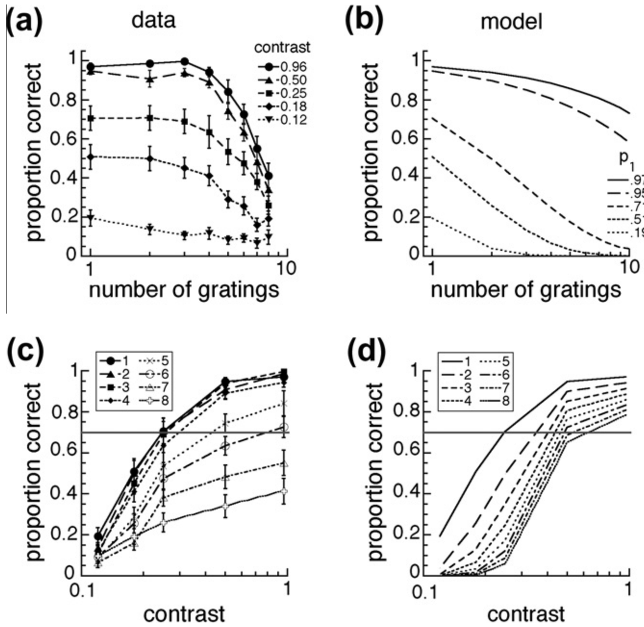

Fig. 2. (a) Actual data. Proportions correct as a function of grating number were better fitted by bilinear functions, although p_1 in a and b were identical. (b) Probability model. Proportions correct as function of grating number predicted by a simple detection model, p_1^n, where p_1 is the probability of seeing 1 grating when 1 grating was presented and n is the number of gratings presented. (c and d) Data as psychometric functions. Proportion correct as a function of contrast for actual and model data. Gray horizontal line indicates 70% correct from which contrast thresholds were derived (see Fig. 3b). $N = 18$, error bar ± SEM.

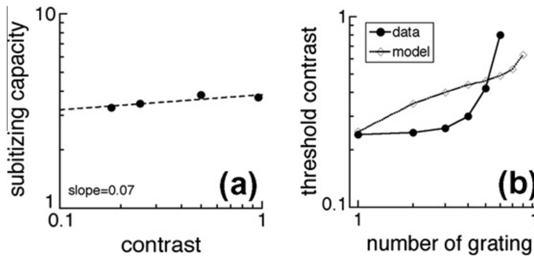

Fig. 3. (a) Subitizing capacity as a function of contrast is between 3 and 4 items. (b). Threshold contrast from psychometric functions of real data also shows a subitizing capacity between 3 and 4 items. Thresholds were taken at 70% correct from psychometric functions seen in Fig. 2c and d.

lowing sections, the data show that enumeration does not completely follow probability summation. While enumeration variability is modified by contrast, subitizing capacity is independent of contrast. These results are consistent with the idea that visual enumeration involves two processes: component detection and component integration, which were differentially affected by component visibility.

3.1. Enumeration accuracy and contrast

We first analyzed how contrast affected enumeration accuracy, and compared it to a probability summation model. We also tested whether discontinuity between small and large numbers, or subitizing capacity, is preserved across contrast. We plotted proportion correct as a function of number for each contrast (Fig. 2a) for 1–8 gratings. Following the method and logic of Green and Bavelier (2006) we estimated subitizing capacity by fitting our data with bilinear functions, with the exception of the lowest contrast used. The first line was set to have a near zero slope, while the second line was set to have a steeper slope. The best bilinear function was found by minimizing the least square error. The intersection between the two functions was taken to be the subitizing capacity (Green & Bavelier, 2006),

We found that subitizing capacity is independent of contrast over a fivefold range of contrasts from 0.96 to 0.18, while maximum proportion correct (i.e., the accuracy at the presentation of 1 grating) changes with contrast. These results suggest that enumeration does not follow the rules of probability based on detecting a single grating (Fig. 2b). Subitizing capacities were 3.71, 3.82, 3.45 and 3.29 at 0.96, 0.50, 0.25 and 0.18 contrast, respectively (Fig. 3a; mean = 3.57 gratings). Watson, Maylor, Allen, and Bruce (2007) found a similar result in a reaction time study in which they varied color similarity between to-be-enumerated target stimuli and to-be-ignored distractor stimuli.

We also carried out an 8 (number) × 5 (contrast) repeated measures ANOVA, that confirmed what is evident in Fig. 2a. We found that proportion correct decreased with decreasing stimulus contrast, $F(4, 68) = 110.82$; $p < 0.0001$, $MSe = 0.106$, We also found that proportion correct decreased with increasing target number, $F(7, 119) = 64.58$; $p < 0.0001$; $MSe = 0.033$, Finally, the interaction between contrast and target number was also significant, $F(28, 476) = 6.87$; $p < 0.0001$, $MSe = 0.020$, suggesting that the effect of contrast varied across target number. We conducted simple planned comparisons of proportion correct at each number to the proportion correct at 1 grating. Across all contrasts, we found that proportions correct at 2 and 3 gratings were not significantly different from the proportion correct at 1 grating ($p > 0.05$), while proportions correct at 5, 6, 7 and 8 patches were significantly different ($p < 0.05$). Proportions correct at four gratings were significantly different from the proportion correct at one grating at 0.96, 0.18 and 0.12 contrasts ($p < 0.05$). These results are consistent with the notion that subitizing, the precise enumeration of <3–4 items, is distinct from counting.

Our data differ somewhat from those of Hunter and Sigler (1940) in that for low luminance dots, enumeration accuracy as a function of element numerosity seems to be better predicted by the probability model (see Schlosberg, 1948). Moreover, their data do not have a constant subitizing capacity of 3–4 items as indexed by this function. This distinction is likely due to the difference in stimulus parameters between these studies such as viewing eccentricity and luminance.

Psychometric functions for enumeration were evaluated by replotting the data as proportion correct as a function of contrast for every number of grating presented (Fig. 2c and d) and deriving threshold contrasts for each number (Fig. 3b). Thresholds were determined by taking the contrast that corresponds to 70% correct (gray line, Fig. 2c and d). Threshold contrasts were flat until 3–4 items and increase thereafter. These data suggest that similar contrasts were required to enumerate 1, 2, 3 or 4 items. Thus, both accuracy (Fig. 3a) and threshold data (Fig. 3b) were consistent with the existence of a subitizing capacity.

Notably, recent evidence from studies of the attentional blink (Egeth et al., 2008; Olivers & Watson, 2008), divided attention (Burr et al., 2010) and inattentional blindness (Railo et al., 2008) shows that attentional resources limit subitizing capacity. The current data shows that subitizing capacity is not limited by stimulus visibility. Together, these results suggest that subitizing capacity is not affected by the general difficulty of the task. This finding is consistent with the classic distinction between data and resource limits (Norman & Bobrow, 1975), in which the effect of attention is best construed as a resource limit, and the effect of contrast as a data limit on visual enumeration.

3.2. Enumeration variability and contrast

To see how enumeration precision and variability changes with contrast, we also evaluated the mean and standard deviation of the responses as a function of number of dots at each contrast for each observer. Fig. 4 represents these data for decreasing element contrast from left to right. Mean responses as a function of number (Fig. 4, first row) generally followed a unit slope for high contrast elements but deviated from unity for lower contrast elements. Slopes were 0.93, 0.96, 0.84, 0.82 and 0.51 at 0.96, 0.50, 0.25, 0.18 and 0.12 contrasts, respectively (Fig. 4a–e). These data show that observers underestimated numerosities as contrast decreased. However observers tended to overestimate the number of elements when only 1 element was presented.

Standard deviations of the responses were calculated for each observer and the average standard deviation was plotted against the number of gratings (Fig. 4, second row). For high contrast stimuli (0.96 and 0.50) variability decreases up to a numerosity of about 2 or 3, and then increases for larger numerosities. For lower contrast levels there is a slight increase in standard deviation with increasing numerosity.

The ratio of the standard deviation and mean of the responses, the coefficient of variation, is often computed in enumeration studies to determine whether variability scales with the represented numerosity (Cordes et al., 2001). If this function has a slope of zero, then it would mean that response variability scales with the number of items (Appendix A). If this function has a log–log slope of -0.5, then it would mean that response variability improves with the number of items, according to the statistical prediction that variance decreases as a square root of number (Appendix B). Except for enumeration at the highest contrast (Fig. third row, first column), coefficient of variance plotted against the number of gratings resulted in negative log–log slopes (between -0.26 and -0.59, Fig. 4, third row). This would suggest that variance scales with number at high contrast (at 0.96), but decreases with number at lower contrasts.

While the variation coefficient function is plotted on log–log coordinates to determine slopes that correspond to a specific hypothesis (see Cordes et al., 2001), many enumeration studies have also used linear–linear coordinates (see (Revkin et al., 2008; Vetter, Butterworth, & Bahrami, 2008). Notably on linear–linear coordinates (Fig. 4, fourth row), these functions displayed discontinuities between the subitizing and counting range across all contrasts except at 0.12 contrast, which planned polynomial contrasts verified (p-values < 0.05). The data from this analysis are parallel to the accuracy data described earlier (Fig. 2a).

We also plotted the histogram of responses regardless of the actual number of gratings presented (Fig. 4, fifth row). The gray bars represent the frequency of the actual element number, and the black bars represent the frequency of the responses. At high contrasts, the histograms of the responses were similar to the histogram of the actual number of gratings – consistent with a uniform distribution (Fig. 4, fifth row, first column). However as visibility decreased, the responses followed a non-uniform distribution with a mode at 2 or 3 items (Fig. 4, fifth row, fourth and fifth column). The means of the response distributions were 4.42, 4.33, 4.33, 3.63 and 2.77 at 0.96, 0.50, 0.25, 0.18 and 0.12 contrasts, respectively. These response frequency data show that while responses trended to lower numerosities as element visibility decreased, the central tendencies of those responses hovered near the veridical mean of the presented stimuli, which was 4.5 elements.

3.3. Enumeration and detection

It is intriguing that enumeration accuracy does not follow the rules of probability because the task of simple detection does. In simple detection, the observer is asked to detect the presence or absence of elements, *p (detect at least 1 element|n elements presented).* Elegant studies of gratings have shown that grating detection is mediated by the parallel activation of independent channels tuned to the characteristics of the grating signal such as spatial frequency (Campbell, 1980; Campbell & Robson, 1968). These studies show that simple detection is governed by the summation of each channel's independent probability of detecting the grating (Graham, 1989). Robson and Graham (1981) found that probability summation reliably depicts improved contrast sensitivities for detecting multiple gratings, particularly in the peripheral visual field. In this context, probability summation means that the more items are presented in a display, the more likely that at least one item is detected. The fundamental principle behind probability summation is that the independent detectabilities of individual elements predict the simultaneous detectability of the whole stimulus. Surprisingly, enumeration – a seemingly slight variation of a detection task – does not completely follow this principle.

However, enumeration does *partially* follow probability summation. The near log–log slope of -0.5 in the coefficient of variation in the responses suggests that probability summation is playing a role in enumeration. These data show that the variability of the responses decreased roughly as the square root of the number gratings. This characteristic in the enumeration functions supports the notion that increasing the number of elements increases overall information available to the observer such that estimation of numerosity improves, in the sense of becoming more precise (i.e., more tightly clustered) with growing numerosity. (For more details, please see Appendix B.)

These data are actually consistent with the notion that there are two core number systems: an approximate number system to represent numerical magnitudes and a precise number system to represent discrete small number of elements (Dehaene, 2009; Feigenson et al., 2004). Here, probability summation is related to the approximate number system, whereas subitizing capacity characterizes the precise number system. On the one hand as luminance contrast decreased, enumeration functions become more in line with numerical magnitude representation. Lower contrast elements showed no discontinuity in the coefficient of variation between 3 and 4 items in log–log coordinates (Fig. 4, third row).

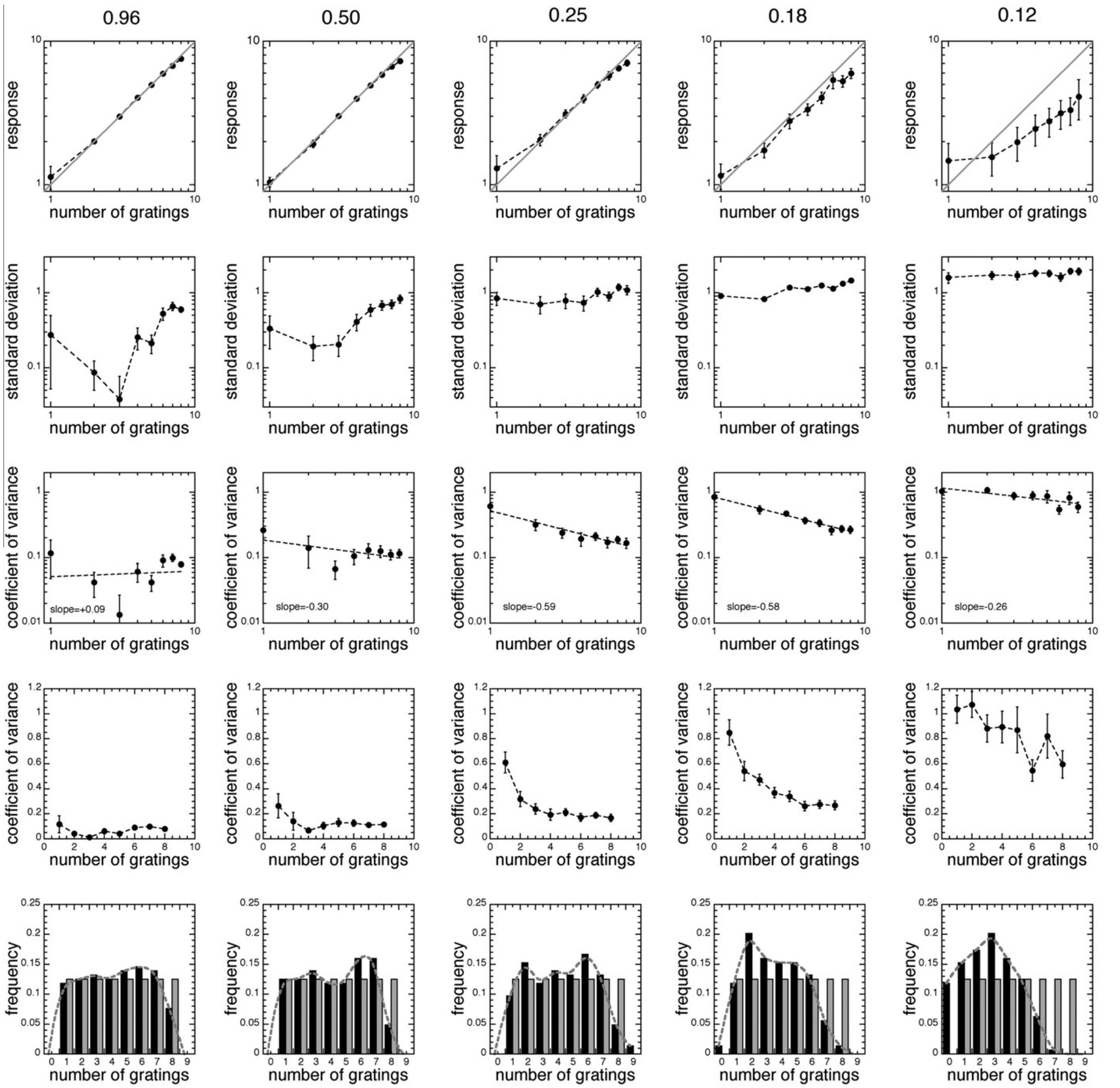

Fig. 4. Characteristics of the enumeration responses from high contrast to low contrast (left to right). (First Row) Mean responses as a function of the number of gratings deviate from a unit slope as gratings become less visible. (Second Row) Standard deviation of responses flattens out against number of gratings, and increases with less contrast. (Third Row) Coefficient of variation functions (standard deviation/mean response) change in slope as contrast decrease in log–log coordinates. (Fourth Row) Coefficient of variation functions in linear–linear coordinates. (Fifth Row) Response histograms (black bars) become less uniform at low contrasts.

Probability summation was most clearly demonstrated in the mean responses at 0.12 contrast (Fig. 4, first row, fifth column), where responses systematically increased with the number of gratings, albeit imprecisely. At this low contrast level, the observers were able to determine which trials had relatively more or fewer elements without knowing the exact number of elements. On the other hand, subitizing capacity was generally unaffected by luminance contrast. Enumeration across multiple contrasts shows discontinuities in the accuracy functions (and variation coefficient functions in linear–linear coordinates). These data suggest that a contrast-independent subitizing capacity operates after element detection, presumably after the process of probability summation of independently detected elements.

3.4. Enumeration and identification

Many aspects of the current data cannot be explained by the simple application of probability theory. This suggests that enumeration involves some process beyond the detection of individual elements. If the computation underlying enumeration is unlike that of simple detection, then what process does it involve? We speculate that enumeration, particularly for small numbers, is like pattern identification, which involves integration of components after detection has occurred. Although identification typically requires integration of contiguous components within an object while enumeration requires integration of spatially separate elements, these tasks share many characteristics such as (1) a similar cortical network, (2) the effect of grouping and spatial geometry, (3) the independence of contrast, and (4) the presence of capacity limits.

The task of identification requires that multiple components are detected and integrated into a recognizable object (Pelli, Burns, Farell, & Moore-Page, 2006; Pelli et al., 2004). In identification, observers track the relative positions of components so that object shapes are correctly identified. For example, the lower-case letters, *b*, *d*, *p* and *q* all have the same components that are in different relative positions. In enumeration, perhaps it is also necessary to implicitly encode the relative position of elements so that the observer can count every item once. Findings from fMRI has shown that both enumeration of dots and discrimination of inferred shape from dots present activations in the posterior parietal cortex (e.g., Ritzl et al., 2003), suggesting that enumeration and shape formation may share a cortical locus.

Enumeration has been linked to shape and structure of element arrays. Mandler and Shebo (1982) proposed that quick enumeration of items is shape formation, in which observers treat the items as canonical patterns such as polygons. Enumeration of elements arranged in well-learned canonical patterns has been found to be faster than enumeration of elements forming irregular patterns (Wender & Rothkegel, 2000). Likewise, Logan and Zbrodoff (2003) also proposed that pattern similarity might account for the subitizing and counting dichotomy. Although the difference between subitizing and counting exists in studies in which polygon formation was prevented by aligning targets in a row (Atkinson et al., 1976; Parth & Rentschler, 1984) shape formation may be related to perception of pattern size, length or regularity as well as perception of polygon characteristics. Burgess and Barlow (1983) found that increasing regularity in the spatial arrangement of dots in a display decreased observer variance in number discrimination while changing regularity in average density and average luminance had little effect. Spatial irregularities in dot arrays have been reported to cause underestimation of numerosity (Ginsburg, 1976, 1978, 1980; Ginsburg & Goldstein, 1987; see also Taves, 1941). It has been proposed that arrays with elements that form sub-clusters tended to be perceived as less numerous, such as in the solitaire illusion (Frith & Frith, 1972) and in randomly distributed dot arrays (e.g., Ginsburg, 1980). More recently, arrays with elements connected by lines have been found to be underestimated as well (Franconeri, Bemis, & Alvarez, 2009; He, Zhang, Zhou, & Chen, 2009). Together, these data suggest that Gestalt grouping cues interacts with enumeration, and is consistent with an enumeration model that segments the output of detected items (Dehaene & Changeux, 1993).

The current data show that subitizing capacity is generally independent of contrast suggesting that enumeration involves component integration, a process distinct from independent detection of individual elements. The recognition of patterns composed of random dot stimuli has been found to be generally contrast invariant as well. These have been demonstrated with stimuli that require spatiotemporal integration of individual dots to detect coherent motion (Burr & Santoro, 2001) or spatial integration of dot pairs to detect Glass patterns (Palomares, Pettet, Vildavski, Hou, & Norcia, 2010). Neurons in primary visual cortex (V1) detect simple local features whereas neurons in extrastriate cortex are tuned to more complex stimuli from the integrated output of V1 neurons. Neuroimaging studies show that responses in V1 is modulated by contrast whereas responses in extrastriate cortex primarily are not (e.g., Tootell et al., 1998.) Thus functions invariant to contrast are likely mediated by mechanisms after the feature detection stage that occurs in V1.

More critically, both enumeration and identification have capacity limits. In enumeration, the subitizing capacity of 3–4 items (Fig. 2b and c) marks the number of objects that can be precisely and quickly enumerated. In identification, the application of the probability summation model to letter identification allows the estimation of the number of components detected at the identification threshold (see Appendix B of Pelli et al., 2006). Pelli et al. (2006) found that observers detected 7 ± 2 components at the identification threshold, across the alphabets tested. They also found that efficiencies for detecting and identifying letters dropped as a function of perimetric complexity suggesting that letters from simple alphabets are more recognizable than letters from complex alphabets. Moreover for identifying letters made up of gratings, efficiency[1] drops in inverse proportion to the number of gratings. Identifying letters made up of 3–4 gratings had an efficiency of 10%, while identifying letters made up of >4 gratings had 2–6% (Majaj et al., 2000), suggesting that patterns comprised of fewer elements are better recognized than patterns with many elements. It is not clear if and how the capacity limits in enumeration and identification are directly related, but these limits may represent the Gestalt idea that simplicity is a "law" of good form (Wertheimer, 1923), such that simple configurations and objects are more readily perceived as a whole. Eye movement data also has a discontinuity between the subitizing and counting range (Watson, Maylor, & Bruce, 2007), with minimal number of saccades in the subitizing range suggesting parallel processing of items.

Interestingly, capacity limits in enumeration have also been linked to capacity limits in visual working memory (see Cowan, 2001 and subsequent discussions). However, storage in working memory is tied to Gestalt grouping principles. Objects that were perceptually grouped have been found to be more likely stored in working memory (Woodman, Vecera, & Luck, 2003). Perhaps, objects within the subitizing capacity can be stored more readily in memory because these objects are more easily grouped, following the Gestalt law of simplicity or Pragnanz (Wertheimer, 1923).

3.5. Enumeration beyond vision

It is observable from counting beans (Jevons, 1871) or speckles on a hen (Butterworth, 2008) and other examples that we have a visual sense of number (Burr & Ross, 2008; see also Durgin, 2008). In this paper, we discussed how the perception of number is related to Gestalt principles of integration, which operate after the detection of independent components. While we focused on how enumeration in the modality of vision, we also acknowledge that the concepts of number and numerosity has been studied in other sensory modalities. Subitizing capacities have been reported for touch (Riggs et al., 2006) and hearing (Camos & Tillmann, 2008), modalities that also possess Gestalt grouping concepts (Harrar & Harris, 2007; Jackendoff and Lerdahl, 2006; Kubovy & Van Valkenburg, 2001).

4. Conclusion

By parametrically varying luminance contrast in an enumeration task, we found that important characteristics of enumeration functions were generally independent of contrast: subitizing capacities were between 3 and 4 items, while the log–log slopes of variation coefficients were near −0.5. Notably, evidence for a subitizing capacity (i.e., discontinuity in accuracy) was absent at the lowest contrast (0.12), whereas evidence of probability summation was absent (i.e., the slope of the variation coefficient was near zero) at the highest contrast (0.96). These results are consistent with the dichotomy between the precise and approximate

[1] Efficiency is the ratio of thresholds between human and ideal observers. Ideal thresholds are dependent on the probability and covariance of the targets as well as the amount of visual noise masking the target (Appendix A, Pelli et al., 2006).

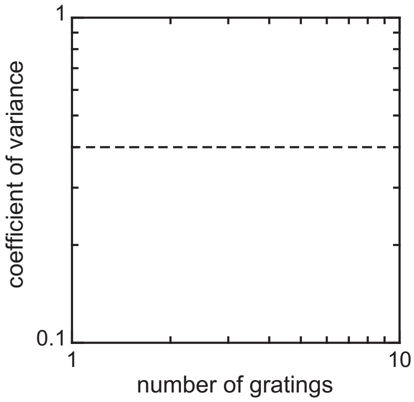

Fig. A1. Scalar variability predictions for theoretical data with k = 0.40. Coefficient of variation as a function of number has a log–log slope of 0 (dotted line).

number systems such that a subitizing capacity represents the limitation in precise enumeration while probability summation of detected elements represents approximate enumeration. In other words, the estimated numerosity of an array reflects the visibility of the individual elements, whereas subitizing does not. We posit that enumeration is linked to the principles of perceptual grouping because both involve integration, beyond summation, of detected componential features.

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Appendix A. Scalar variability model of enumeration

Coefficient of variation is defined as the standard deviation divided by the mean of the response. This is a normalized metric of enumeration precision (see Cordes et al., 2001) identical to the concept of Weber fraction (Piazza et al., 2002; Nieder & Merten, 2007). If enumeration precision scales with element number, then the variation coefficient would be constant (k) as a function of number (n).

$$\sigma = kn \tag{A1}$$

$$\text{coefficient of variation} = k \tag{A2}$$

In this case, standard deviation increases with element number, but normalized standard deviations do not. Fig. A1 shows an example where k = 0.4.

Appendix B. Binomial variability model of enumeration

The probability model of summation described in the main text is based on the binomial distribution, a discrete probability distribution of the number of successes that individual elements were detected. The probability of exact enumeration (p) depends on the probability of detecting one element (p_1) and the number of elements (n) according to the following relationship $p = p_1^n$ (see Section 1.1).

The mean and variance of the binomial distribution is also predicted by the probability of detecting one element (p_1) and the number of elements (n). The mean of the binomial distribution is:

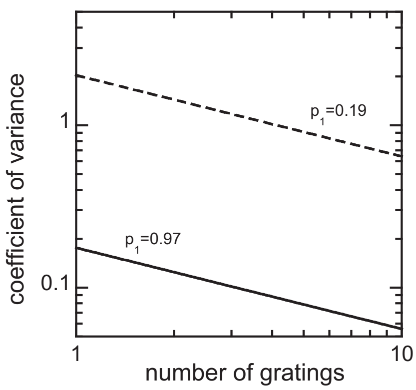

Fig. B1. Binomial variability predictions for theoretical data for p_1 = 0.97 (solid line) and p_1 = 0.19 (dotted line). Coefficient of variation as a function of number has a log–log slope of -0.5, regardless of the value of p_1.

$$\mu = np_1 \tag{B1}$$

The variance of the binomial distribution is:

$$\sigma^2 = np_1(1 - p_1) \tag{B2}$$

Standard deviation is the square root of the variance. Thus the standard deviation of the binomial distribution is:

$$\sigma = (np_1(1 - p_1))^{1/2} \tag{B3}$$

Since the coefficient of variation is the standard deviation divided by the mean, the coefficient of variation for a binomial distribution is:

$$\text{coefficient of variation} = ((np_1(1 - p_1))^{1/2})/np_1 \tag{B4}$$

Simplified:

$$\text{coefficient of variation} = ((1 - p_1)/np_1)^{1/2} \tag{B5}$$

Put in a familiar form:

$$\text{coefficient of variation} = (np_1/(1 - p_1))^{-1/2} \tag{B6}$$

Note that the exponent is $-1/2$ (or -0.5), which corresponds to the slope in log–log coordinates. Fig. B1 shows examples of coefficient of variation as a function of element number.

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